Finite temperature effective actions for double wells

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Abstract

The problem of calculating the effective potential for a real scalar field with a double–well potential is a possible preliminary to understanding symmetry breaking phase transitions in the early universe. It is argued here that it is necessary to include multiple saddle points in the path integral formalism and that quadratic source terms are important. The resulting effective action has a well–defined propagator expansion with a positive mass and gives a new way of understanding high temperature expansions, but the results are equally valid at low or zero temperatures.

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I. INTRODUCTION

The favoured models of particle physics and cosmology imply that the hot material of the early universe underwent a series of phase transtions where the nature of the fundamental forces changed. These transitions are associated with the breaking of a symmetry of the dynamical equations due to a Higgs field developing an expectation value in the lower temperature phase.

The physics of these high temperature phase transitions is often described by an effective potential for the expectation value of the Higgs field [1–4]. The definition of this potential for a real scalar field ϕ and the symmetry group Z_2 is the issue with which we shall be concerned in what follows.

In order to break the symmetry the classical potential should have a global minimum away from the origin, as shown in figure 1. In applications concerning the early universe we are particularly concerned with the way in which the field leaves the origin and approaches the global minimum. This can be described by using a quantum effective potential $V_\beta(\phi)$, but some care has to be taken in defining it. A simple loop expansion fails because the mass is imaginary at the classical level, and the usual proceedure would be to use some form of 'corrected' mass, related to resummation of subsets of the graphs in the loop expansion.

The definition of the quantum effective potential $V_\beta(\phi)$ is even more difficult in a system which has a finite volume, a situation which arises when cosmological effects are important. Then $V_\beta(\phi)$ can be shown to be convex [5]. The underlying reason for the convexity is that the ground state is a superposition of wave functions $\Psi_\pm$ that are peaked about the homogeneous field values $\phi_\pm$ at the minima of the potential. In a path integral construction of the effective potential there are two equally dominant field configurations [6]. When $V_\beta(0)$ is evaluated it is dominated by both minima of the classical potential and tells us nothing about the behaviour of the system near to the symmetric point $\phi = 0$.

Some years ago it was suggested that a more satisfactory approach to the finite temperature effective action [7,8] was to follow Cornwall, Jackiw and Tomboulis [9] and introduce a source $K(x, x')$ coupled to quadratic terms in the action. The reason for the effectiveness of this source is that it is able to prepare the quantum state of the field to be concentrated near to the symmetric value, the place it would occupy before the onset of a supercooled phase transition. This would be similar to using a corrected mass in the usual approach, but would apply equally to extreme supercooling where $T = 0$ as to high temperatures.

In the formalism of reference [9], the effective action is a function of the field expectation value $\phi(x)$ and the connected propagator $G(x, x')$. The physical solutions satisfy

$$\frac{\delta \Gamma[\phi, G]}{\delta \phi(x)} = J(x) - \int d\mu(x') K(x, x') \phi(x'), \tag{1}$$

$$\frac{\delta \Gamma[\phi, G]}{\delta G(x, x')} = -\tfrac{1}{2} K(x, x'). \tag{2}$$

If the quadratic source is set to zero then we can solve the second equation for $G[\phi]$. The result would be the usual effective action,

$$\Gamma[\phi] = \Gamma[\phi, G[\phi]]. \tag{3}$$

Amelino–Camelia and So–Young Pi [10] have shown that this approach provides an efficient way of resuming the loop diagrams to obtain the corrected mass.

The new idea that will be used here is that other linear combinations of 1 and 2 can be solved for $G[\phi]$ to obtain different effective actions $\Gamma[\phi, G[\phi]]$. From the different possibilities, one is chosen that satisfies the following criteria:

1. The effective action generates effective field equations,

$$\frac{\delta \Gamma[\phi]}{\delta \phi} = 0 \tag{4}$$

2. It has a well–defined loop expansion.
3. It reduces to the calssical action as $\hbar \to 0$,

$$\Gamma[\phi] = I[\phi] + O(\hbar) \tag{5}$$

When the mass is imaginary it is possible to obtain an effective potential this way which has a symmetry breaking minimum in the infinite volume limit. The field can then evolve smoothly or tunnel through the potential from the symmetric point. At high temperatures the potential is similar to the usual potential with a corrected mass, but the method gives a uniform treatment of low as well as high temperatures.

In constructing the effective action for finite volumes it is necessary to allow several field configurations to dominate the path integral simultaneously. Near to the symmetric value of the field at $T < T_c$ any linear source terms become very small and the quadratic source terms are the most important. There is some similarity here to work by Laurie [11] and Cahill [12], who use quadratic sources but do not take into account the effects of multiple saddle points. Beyond the minima of the potential, the linear sources take over and the potential reverts to the usual form.

The inverse of $G(x, x')$ is the quantum fluctuation operator for the Higgs fields. With the construction described here the operator comes out to be positive definite. Furthermore, the potential has a barrier around the origin which has a different height than for some versions of the temperature corrected potential (e.g. [2]), but since differences can arise from resumming different sets of graphs these differences may not be significant. For high temperature field theory the new approach can be viewed as an attempt to justify the use of corrected masses by introducing a quadratic source in the path integral.

II. DOUBLE WELLS AT ZERO TEMPERATURE

This section begins by setting up the definition of the effective action as a function of composite fields following reference [9]. We then go on to consider the modifications that are necessary when there is more than one field configuration dominating the path integral. A Riemannian signature $(++++)$ will be used and units in which the velocity of light $c = 1$.

The classical action for a real scalar field will be taken to be of the form

$$I[\phi] = \int d\mu(x) \left\{ -\tfrac{1}{2} \nabla \phi . \nabla \phi + V(\phi) \right\}, \tag{6}$$

with a classical double well potential

$$V(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^2. \tag{7}$$

The generating function $Z[J, K]$ is defined by a path integral,

$$Z[J, K] = \int d\mu[\phi] e^{-I_{JK}[\phi]/\hbar} \tag{8}$$

where

$$I_{JK}[\phi] = I[\phi] + \int d\mu(x) J(x) \phi(x) +$$
$$\tfrac{1}{2} \int d\mu(x, x') \phi(x) K(x, x') \phi(x') \tag{9}$$

If we set $W = -\hbar \ln Z$, then the expactation values ϕ and connected propagators G are

$$\phi(x) = \frac{\delta W}{\delta J(x)} \tag{10}$$

$$G(x, x') = 2\frac{\delta W}{\delta K(x, x')} - \frac{\delta W}{\delta J(x)} \frac{\delta W}{\delta J(x')} \tag{11}$$

The effective action is defined by eliminating J and K in favour of ϕ and G,

$$\Gamma[\phi, G] = W[J, K] - \int d\mu(x) \phi(x) J(x)$$
$$-\tfrac{1}{2} \int d\mu(x, x') \phi(x) K(x, x') \phi(x') - \tfrac{1}{2} \int d\mu(x, x') G(x, x') K(x, x'). \tag{12}$$

Before proceeding further it should be instructive to examine the nature of the field configurations ϕ_s that dominate the path integral. This can be simplified to the case of homogeneous fields, for which

$$J(x) = j, \quad K(x, x') = k\delta(x - x') \tag{13}$$

with constants j and k. The saddle point condition,

$$\frac{\delta I_{JK}}{\delta \phi} = 0, \tag{14}$$

then implies that

$$V'(\phi_s) + j + k\phi_s = 0. \tag{15}$$

Solutions are sketched in figure 2. There are three saddle points when $j^2 < 4(\mu^2 - k)^3/27\lambda$, one of which has a much larger action than the other two and can usually be neglected. There is a series expansion for the solutions in powers of j,

$$\phi_{s\pm} = \pm\frac{(\mu^2 - k)^{1/2}}{\sqrt{\lambda}} - \tfrac{1}{2}(\mu^2 - k)^{-1}j + O(j^2). \tag{16}$$

The generating function Z can be evaluated by shifting the fields and constructing an asymptotic expansion in $\hbar$ around each of the saddle points seperately. The two series have then to be summed,

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$$Z \sim e^{-W_+/\hbar} + e^{-W_-/\hbar}. \tag{17}$$

If we set $\phi = \phi_\pm + \tilde{\phi}$, then the asymptotic expansions are generated by

$$e^{-W_\pm[J,K]/\hbar} = e^{-I_{JK}[\phi_\pm]/\hbar}$$
$$\int d\mu[\tilde{\phi}] \exp \frac{1}{\hbar} \left\{ -\int \tilde{\phi}(\Delta_\pm + K)\tilde{\phi} - \int J_{int}\tilde{\phi} - I_{int} \right\}. \tag{18}$$

The interaction term I_{int} consists of the terms from $I[\phi_\pm + \tilde{\phi}]$ that are cubic or higher order in $\tilde{\phi}$. The other terms are,

$$J_{int}(x) = \frac{\delta I_{JK}}{\delta \phi(x)} \tag{19}$$

$$\Delta(x, x') = \frac{\delta^2 I}{\delta \phi(x) \delta \phi(x')} \tag{20}$$

Instead of choosing solutions of the classical equations for $\phi_\pm$ it is better to use quantum corrected background fields. Define local quantum expectation values of the field operator by

$$\langle \phi \rangle_\pm = \frac{\delta W_\pm}{\delta J}, \tag{21}$$

then the fields $\phi_\pm$ will be defined by

$$\phi_\pm = \langle \phi \rangle_\pm. \tag{22}$$

Because of 14, $\phi_\pm = \phi_{s\pm}$ to leading order in $\hbar$.

The expectation value of the field ϕ is given by the generating function,

$$\phi = Z^{-1} \frac{\delta Z}{\delta J}. \tag{23}$$

From the asymptotic expansion 17,

$$\phi \sim \frac{e^{-W_+/\hbar}}{Z} \phi_+ + \frac{e^{-W_-/\hbar}}{Z} \phi_- \tag{24}$$

Some re–arrangement allows this to be written as

$$\phi \sim \tfrac{1}{2}(\phi_+ + \phi_-) - \tfrac{1}{2}(\phi_+ - \phi_-)\,\Theta, \tag{25}$$

where

$$\Theta = \tanh \left(\frac{W_+ - W_-}{2\hbar} \right). \tag{26}$$

The propagator can be expressed likewise. We set

$$\hbar G_\pm(x, x') = \langle \tilde{\phi}(x)\tilde{\phi}(x') \rangle_\pm. \tag{27}$$

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From the definition of G, equation 11, we get

$$\hbar G(x, x') \sim \frac{e^{-W_+/\hbar}}{Z}\hbar G_+(x, x') + \frac{e^{-W_-/\hbar}}{Z}\hbar G_-(x, x') +$$
$$\frac{e^{-(W_+ + W_-)/\hbar}}{Z^2}(\phi_+(x) - \phi_-(x))(\phi_+(x') - \phi_-(x')). \tag{28}$$

We write this in the form,

$$\hbar G \sim \tfrac{\hbar}{2}(G_+ + G_-) - \tfrac{\hbar}{2}(G_+ - G_-)\Theta + \rho^2, \tag{29}$$

where

$$\rho^2(x, x') = \tfrac{1}{4}(1 - \Theta^2)(\phi_+(x) - \phi_-(x))(\phi_+(x') - \phi_-(x')). \tag{30}$$

The meaning of these expressions can be made clearer by returning to the homogeneous case. To leading order in $\hbar$ we replace $\phi_\pm$ in equation 25 by the classical solution $\phi_{s\pm}$ (equation 16),

$$\phi = -\phi_0 \tanh(\Omega\phi_0 j/\hbar) - \tfrac{1}{2}(\mu^2 - k)^{-1}j + O(j^2) \tag{31}$$

where $\phi_0(k)$ is the value of the classical solution at $j = 0$ and Ω is the spacetime volume. Evidently, ϕ covers the whole range of values from $-\phi_0$ to ϕ_0 over a very small range of j. This is the first indication that we should set $j \to 0$ as $\Omega \to \infty$.

In the same approximation, the propagator contains the term ρ given by,

$$\rho^2 = (1 - \Theta^2)\phi_0^2 + O(j^2) = \phi_0^2 - \phi^2 + O(j) \tag{32}$$

If we chose a form for G where $\rho = 0$ then we must have $\phi \sim \phi_0$, i.e. the value of ϕ is set by the quadratic source. (This condition on ρ was implicitly assumed in ref [10], although it holds automatically only as long as the mass is real.)

We turn now to the effective action. For the double saddle 17,

$$-\hbar \ln Z \sim \tfrac{1}{2}(W_+ + W_-) + \tfrac{1}{2}\hbar \ln(1 - \Theta^2) \tag{33}$$

We can construct an asymptotic series for W_+ in terms of Feynmann diagrams with propagator $G_0 = (\Delta + K)^{-1}$. The contributions that are two–loop and higher will be labelled $W_+^{(2)}$,

$$W_+ \sim I_{JK}[\phi_+] + \tfrac{1}{2}\hbar \ln \det(\Delta_+ + K) + W_+^{(2)}. \tag{34}$$

Two conditions will be imposed, prompted by the discussion of the homogeneous case. First of all, the condition that $\rho \to 0$ as $\Omega \to \infty$ will be imposed on G. This guarantees that the propagator has a simple local form to leading order in $\hbar$. It also forces the state of the system away from the global minima of the potential when ϕ is small. It follows from equation 30 that $\phi_+ \to \phi_-$ or $\Theta \to \pm 1$. The first of these would correspond to the situation which applies to single well potentials. In the second case, the condition forces only one of

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the two saddles to dominate. If $\Theta \to -1$, then it follows from 25 that $\phi \to \phi_+$, from 29 that $G \to G_+$ and from 31 that $\Omega J \to -\infty$.

The second condition is that $J \to 0$ as $\Omega \to \infty$ (taking $J \sim \Omega^{-1/2}$ for example). This condition allows us to solve for $K[\phi]$, or equivalently $G[\phi]$. If we chose $K \to 0$ instead, then G would be a propagator with an imaginary mass in the limit $\hbar \to 0$, leading to problems with the effective action at the one–loop level. These problems could be solved by resumming series of graphs, but this is a proceedure which the present approach seeks to avoid. The significance of the two conditions is explained further in the next section.

With these conditions and equation 12 we get,

$$\Gamma[\phi] \sim I[\phi] - \tfrac{\hbar}{2}\text{tr}(GK) + \tfrac{\hbar}{2}\ln\det(\Delta + K) + W^{(2)}. \tag{35}$$

The source K is found by the solution of equation 22 and G is given by equation 27. Diagramatically, equation 22 is equivalent to setting the sum of all tadpole diagrams in figure 4 to zero. The diagrammatic expansion of the propagator is shown in figure 5.

The effective action can be rewritten also in a more elegant form. First, we separate the one–loop and higher part $G^{(1)}$ of G,

$$G = G_0 + G^{(1)}. \tag{36}$$

Then,

$$G_0 = G(1 - G^{-1}G^{(1)}) \tag{37}$$

and

$$K = G_0^{-1} - \Delta = (1 - G^{-1}G^{(1)})^{-1}G^{-1} - \Delta. \tag{38}$$

Whan substituted into equation 35, The propagator corrections have the effect of cancelling all of the two–particle reducible diagrams in $W^{(2)}$. This allows the result to be written in the form,

$$\Gamma[\phi] \sim I[\phi] - \tfrac{\hbar}{2}\text{tr}(1 - G\Delta) - \tfrac{\hbar}{2}\ln\det G + \Gamma^{(2)}, \tag{39}$$

where $\Gamma^{(2)}$ has only two–particle ireducible diagrams in the propagator G. The result now looks the same as the one in reference [9], but it should be realised that the result only takes this simple form in the limit that $J \to 0$.

To leading order in $\hbar$, $G = (\Delta + K)^{-1}$ and K is given by $J_{int} = 0$ (19). Therefore, in the homogeneous case,

$$k = \mu^2 - \lambda\phi^2 \tag{40}$$

and from equation 20,

$$G^{-1} = -\nabla^2 + V''(\phi) + k = -\nabla^2 + 2\lambda\phi^2. \tag{41}$$

There are two remarkable features of this result. The first is that the operator is positive definite even when $V''(\phi)$ is negative. The second is that it differs substantially from the corresponding operator for a single well,

$$G^{-1} = -\nabla^2 + V''(\phi) = -\nabla^2 - \mu^2 + 3\lambda\phi^2. \tag{42}$$

where μ is imaginary.

III. A DOUBLE–POTENTIAL FOR FINITE VOLUMES

The form of the effective action that has been arrived at depends crucially upon the conditions that where imposed on G. It would be instructive to investigate these conditions further by keeping ρ non–zero. Homogeneous fields will be used. Equation 32 for ρ tell us that

$$\phi_\pm = \pm(\rho^2 + \phi^2)^{1/2} + O(j). \tag{43}$$

When substituted into equation 33 for $\ln Z$ this gives

$$-\hbar \ln Z = I_{JK}[\sqrt{\rho^2 + \phi^2}] + \tfrac{1}{2}\hbar \ln(1 - \Theta^2) + O(j^2), \tag{44}$$

with $O(j^2)$ because the action is stationary at ϕ_+. From the definition of the effective action, we have

$$\begin{aligned}
\Gamma &= I_{JK}[\sqrt{\rho^2 + \phi^2}] + \tfrac{1}{2}\hbar \ln(1 - \Theta^2) \\
&\quad -\Omega j\phi - \tfrac{1}{2}\Omega k(\phi^2 + \rho^2) + O(j^2).
\end{aligned} \tag{45}$$

Now we use

$$j = \frac{\hbar}{\phi_0 \Omega} \tanh^{-1} \Theta + O(j^2) \tag{46}$$

from equation 31 and

$$\Theta = -\frac{\phi}{\sqrt{\rho^2 + \phi^2}} + O(j) \tag{47}$$

from equation 32. We define a 'zero–loop' effective potential by $V^{(0)} = \Gamma/\Omega$, and then

$$V^{(0)}(\rho, \phi) = V(\sqrt{\rho^2 + \phi^2}) + V_\Omega(\rho, \phi) + O(j) \tag{48}$$

where

$$V_\Omega(\rho, \phi) = \frac{\hbar}{\Omega}\left\{ \frac{\phi}{\sqrt{\rho^2 + \phi^2}} \tanh^{-1} \frac{\phi}{\sqrt{\rho^2 + \phi^2}} + \tfrac{1}{2} \ln \frac{\rho^2}{\rho^2 + \phi^2} \right\}. \tag{49}$$

This double–potential has been plotted in figure 6.

The global minimum of the double–potential lies at $\phi = 0$ and $\rho = \phi_0$, where $\phi_0^2 = \mu^2/\lambda$. This minimum represents a superposition of states at the two minima $\pm\phi_0$. However, as $\Omega \to \infty$ the mixing between the two minima approaches zero and this is represented by the valley bottom of the potential levelling off allowing the field to remain at any point in the bottom of the valley indefinitely.

Now consider what happens when the field is released from the vicinity of the symmetric point. If ϕ and ρ are very small, then the potential is steepest in the ρ direction and therefore ρ starts to grow. This is the growth in $\langle \phi^2 \rangle$ that has been noticed before in the context of early universe phase transitions [7].

The subsequent evolution of ϕ depends on the initial 'velocity'. If the initial component of velocity in the ρ direction is negligeable and the volume is very large, then the growth in ρ will be tiny and $\rho = 0$ will be a good approximation. Equations for the evolution of ϕ can then be found from variation of the effective action. In the previous section we saw that corrections to the kinetic term are of order $\hbar$, hence

$$- \nabla^2 \phi + V'(\phi) = O(\hbar). \tag{50}$$

These equations result in a 'roll' down the potential hill of the kind that is required by certain inflationary models [13–15].

The same conclusion can be reached by fixing the momentum of the fields in the effective action [16]. This approach has some of the advantages of the present approach, in particular it introduces conditions in which the potential need not be convex.

The old–style effective potential has only a linear source term and can therefore be recovered by setting k to zero and solving for ρ. From equations 14 and 32, this implies that $\rho^2 + \phi^2 = \mu^2/\lambda$. When $\phi^2 > \mu^2/\lambda$, then there is a single saddle point and $\rho = 0$. The result is a convex potential for ϕ, as indeed it should be [5].

IV. DOUBLE WELLS AT FINITE TEMPERATURE

The equilibrium states of a quantum field in a canonical ensemble with temperature $T = 1/\beta$ are located at the minima of the free energy $F = \beta \Gamma$ [2,3], where Γ is the finite temperature analogue of the effective action. For a scalar field at very high temperatures the minimum is unequivocably at the symmetric value of the field. At low temperatures, below a critical value T_c, the predominant quantum states of the scalar scalar field cause the symmetry to be broken. Subject to a suitable solution to the convexity problem, this should be reflected in a minimum of the effective potential $V_\beta(\phi)$ away from the symmetric value.

Below the critical temperature it will become necessary to use the formalism developed for the double well in the previous sections of this paper. The change from zero temperature to the canonical ensemble means that the spacetime should be changed to one that is periodic in imaginary time with period β.

Homogeneous fields will be used once again, and the effective potential defined by

$$\Gamma[\phi] = \Omega V_\beta(\phi) \tag{51}$$

From the previous result for Γ, equation 39,

$$V_\beta = V + V^{(1)} + V^{(2)} + O(\hbar^3), \tag{52}$$

where

$$V^{(1)} = -\tfrac{1}{2}\hbar(G^{-1} - \Delta)G(x,x) - \tfrac{1}{2}\hbar\Omega^{-1}\ln\det G \tag{53}$$

$$V^{(2)} = \tfrac{1}{8}6\hbar^2\lambda G(x,x)^2 \tag{54}$$

An effective 'mass' m can be defined by

$$G^{-1} = -\nabla^2 + m^2 + O(\hbar). \tag{55}$$

(In fact $\hbar m$ has the unit of mass). The propagator is defined by the diagrams in figure 5. The ring representing $G(x, x)$ is of order $\hbar^{-2}$, but at zero temperatures it can be absorbed into the renornalisation of the mass and can be ignored. This is no longer the case at finite temperatures where the rings have to be retained to keep all of the terms up to order $\hbar$ in the propagator. Using the approximation 55, the effect of these rings is to shift the mass m,

$$m^2 = V''(\phi) + k + 3\lambda\hbar G(x, x). \tag{56}$$

The value of k is fixed by the other set of graphs 3, which represent the integrals

$$- \int d\mu(x')G(x, x')J_{int}(x')$$
$$-3\lambda\hbar \int d\mu(x')G(x, x')G(x', x')\phi(x') + O(\hbar^2). \tag{57}$$

Multiplying by $G(x, x')^{-1}$ gives an equation for J_{int},

$$V'(\phi) + k\phi = -3\lambda\hbar G(x, x)\phi. \tag{58}$$

For the scalar field potential,

$$k = \mu^2 - \lambda\phi^2 - 3\lambda\hbar G(x, x). \tag{59}$$

This allows k to be substituted into equation 56,

$$m^2 = 2\lambda\phi^2 + O(\hbar). \tag{60}$$

The ring corrections have cancelled and no longer appear in the mass to this order of approximation.

A high temperature expansion for the functional determinants and the propagator can be used when $T >> \hbar m$ [2,3],

$$G(x, x) \sim \hbar^{-2}\left(\tfrac{1}{12}T^2 - \tfrac{1}{6\pi}\hbar mT\right) \tag{61}$$

and

$$\ln \det G \sim \Omega\hbar^{-4}\left(\tfrac{\pi^2}{45}T^4 - \tfrac{1}{12}\hbar^2 m^2 T^2 + \tfrac{1}{6\pi}\hbar^3 m^3 T\right). \tag{62}$$

Substituted into the effective potential, these give (dropping a constant)

$$V_\beta \sim V + \tfrac{1}{12\pi}(\mu^2 - \tfrac{1}{4}\hbar^{-1}\lambda T^2)Tm + \tfrac{1}{16}\hbar^{-1}T^2 m^2 - \tfrac{1}{8\pi}Tm^3. \tag{63}$$

The mass is given by $m^2 = 2\lambda\phi^2$. The potential is shown in figure 7.

The result is different from the one we would obtain from a single saddle point,

$$V_\beta \sim V + \tfrac{1}{24}\hbar^{-1}T^2 m^2 - \tfrac{1}{12\pi}Tm^3, \tag{64}$$

with the mass given by ring corrections only,

$$m^2 = -\mu^2 + 3\lambda\phi^2 + \tfrac{1}{12}\hbar^{-2}T^2. \tag{65}$$

However, there is agreement on the value of the critical temperature in each case.

The new results are similar to results that can be obtained from a 'self–consistent' corrected mass [4], particularly with the value of $m^2 = 2\lambda\phi^2$. One particular feature is the presence of a linear term. Although these are sometimes seen in the usual approach [17], the authors of reference [18] have argued that they arise from a badly truncated perturbation series. The approach presented here has been trucated after a fixed number of loops, but it is possible to re–order the series, if it is so desired, to truncate the series at a fixed order in $\hbar$, λ or T.

The detailed form of the potential is particularly important in the case of a first order transition, where the field tunnells through a potential barrier to the stable phase. The tunnelling rate is governed by a 'bounce' solution, a solution to the Riemannian (i.e. imaginary time) equations. The bounce solution satisfies equations 1 and 2 and will therefore be at a stationary point of the effective action. The tunnelling rate depends quite sensitively on the form of the potential.

V. CONCLUSIONS

The effective action is a very useful tool for the study of symmetry breaking and phase transitions in the early universe. An attempt has been made in this paper to construct the effective action in a region where the potential is curving downwards as a consistent loop expansion, both at zero or non–zero temperatures and for finite or infinite volume systems.

We have seen that the use of quadratic sources allows a consistent treatment of the effective action and that multiple contributions to the path integral have to be taken into account. The corrected mass emerges naturally rather than being introduced 'by hand'. The effective action has the form,

$$\Gamma[\phi] \sim I[\phi] - \tfrac{\hbar}{2}\mathrm{tr}(1 - G\Delta) - \tfrac{\hbar}{2}\ln\det G + \Gamma^{(2)}, \tag{66}$$

with G given by the diagramatic expansion shown in figure 5. The propagator G_0 which appears in the expansion depends upon the number of saddle points that may contribute to the action,

Case I. A single saddle point, where it is possible to take the quadratic source K to be zero and G_0 is the inverse of the classical fluctuation operator 20.

Case II. Two saddle points, where K is given by the vanishing of the tadpole diagrams in figure 4 and $G_0 = (\Delta + K)^{-1}$.

One or the other case gives a properly defined loop expansion. The second case applies in particular to finite temperature systems below their critical temperature and corresponds to what would normally be understood as the use of a 'corrected mass'.

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FIGURES

FIG. 1. The scalar field double–well potential.

FIG. 2. The classical saddle point solution.

FIG. 3. The vertices of the shifted theory. The lines represent the propagator G_0.

FIG. 4. The tadpole diagram series. Lines with circles represent the connected propagator G, other circles represent connected vertices.

FIG. 5. Iterating this series gives the diagramatic expansion of the propagator G in terms of the shifted propagator G_0.

FIG. 6. The effective potential $V^{(0)}(\rho, \phi)$.

FIG. 7. The finite temperature potential for a range of temperatures. All of the temperatures are less than T_c.